MATHEMATICAL AND COMPUTER MODELING OF NONLINEAR BIOSYSTEMS I

COMPUTER LABORATORY IV: Classic L-V (solutions, average value, harvesting), L-V with logistic term

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Lotka-Volterra model

Consider the following predator-prey system

$$\begin{cases} \dot{V} = rV - aVP\,, \\ \dot{P} = -sP + abVP\,, \end{cases}$$

where all parameters are positive and V and P reflect the size of prey and predator populations, respectively.

From the lecture we know that the solutions to the above system are periodic.

Let us investigate numerically that periodicity.

Exercise 1 - solving the system

Implement MATALB function returning numerical solution to L-V system

$$\begin{cases} \dot{V} = rV - aVP\,, \\ \dot{P} = -sP + abVP\,, \end{cases}$$

for given initial conditions, parameters and time interval.

Your function should accept any number of additional arguments and pass them to the ode solver.

TIP: use the variable-length input argument list (varargin)

Exercise 1 - solution

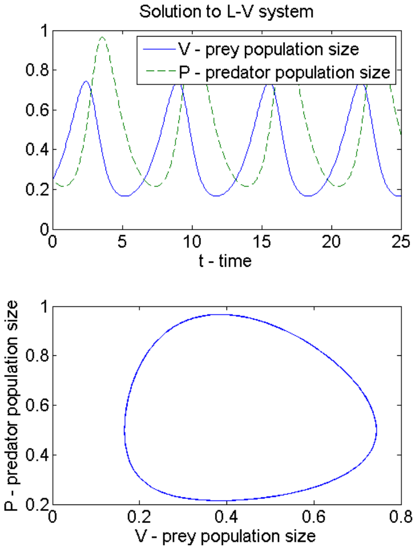
```
function sol=solutionLV(T, init, par, varargin )

    sol = ode45(@model,[0 T],init,varargin{:});

    function y=model(~,x,varargin)
      y=ones(2,1);
      y(1)=par.r*x(1)-par.a*x(1)*x(2);
      y(2)=-par.s*x(2)+par.a*par.b*x(1)*x(2);
    end

end
```

Excercise 2 - plotting L-V

Implement MATLAB function for plotting trajectory of L-V system in a phase plane and its solution in time (use subplot function).

Use different LineStyles, make axis labels, etc.

Accept figure number as an argument.
Plot for $r = 1$, $a = 2$, $s = 1$, $b = 1.3$ and $(V(0, P(0)) = (0.25, 0.25)$.



Exercise 2 - solution

```
function plotLV( sol, f )
    set(0,'DefaultAxesFontSize',16)

    figure(f)
    clf
    subplot(2,1,1)
    pl=plot(sol.x,sol.y);
    set(pl(2),'LineStyle','--');
    xlabel('t - time')
    legend({'V - prey population','P - predator population'})
    title('Solution to L-V system')

    subplot(2,1,2)
    plot(sol.y(1,:),sol.y(2,:));

    xlabel('V - prey population size')
    ylabel('P - predator population size')
end
```

Estimating the period

We know that the solutions to L-V are periodic.

We would like to estimate that period for any given parameters and initial condition.

With the method for period estimation we could see period dependence on the parameters values.

Excercise 3 - estimating the period

Implement MATLAB function that returns the estimation of the period in the L-V system.

Input arguments: initial condition, parameter values.

Use function defined in Ex. 1.

TIP: use the 'Event' option for ode solver.

Exercise 3 - solution

```
function period=periodLV(init, par )

    period = 0;
    opt = odeset('Events',@events,'RelTol',1e-8,'AbsTol',1e-8);
    T=20;
    while ~period
        sol = solutionLV(T,init, par, opt);
        Vevents=sol.xe(sol.ie==1);
        if length(Vevents)>2
           period = Vevents(end)-Vevents(end-2);
        end
        T=2*T;
    end
```

Continues on the next slide

Exercise 3 - solution

```
    function [value,isterminal,direction] = events(~,x)
        value=[par.r*x(1)-par.a*x(1)*x(2);
               -par.s*x(2)+par.a*par.b*x(1)*x(2)];
        isterminal=[0; 0];
        direction=[0; 0];
    end
end
```

Excercise 4 - period dependence

Plot the dependence of period on the prey growth rate (r) and on the predator death rate (s).

Implement single function with the arguments:

- mesh of points for r;
- mesh of points for s;
- parameters values;
- initial conditions.

Exercise 4 - solution

```
function out = plotDependence( rmesh, smesh, par, init )
    n1 = length(rmesh);
    n2 = length(smesh);
    out = zeros(n1, n2);
    for i=1:n1
        for j=1:n2
            par.r=rmesh(i);
            par.s=smesh(j);
            out(i,j)=periodLV(init,par);
        end
    end

    figure(2)
    clf
    surface(smesh,rmesh,out)
    grid on;
    box on;
end
```

Excercise 5 - coprazion with theory

From the lecture we know that the following equalities are true:

$$\bar{P} = \frac{1}{T}\int_{0}^{T} P(t)dt = \frac{r}{a},$$

and

$$\bar{V} = \frac{1}{T}\int_{0}^{T} V(t)dt = \frac{s}{ab},$$

where T is the period of the solution.

Check if they hold numerically.

Exercise 5 - solution

Function retuning theoretical and numerical values of averages

```
function [avNum, avTheo] = averageValues( init, par)

    %theoretical values
    avTheo = [ par.s/par.a/par.b, par.r/par.a];

    %numerical approximation
    period = periodLV(init,par);

    opt = odeset('RelTol',1e-8,'AbsTol',1e-8);
    sol = solutionLV(period,init, par, opt);

    avNum=trapz(sol.x,sol.y')/period;

end
```

L-V system with logistic growth term

Consider the following predator-prey system

$$\begin{cases} \dot{V} = rV\left(1 - \frac{V}{K}\right) - aVP\,, \\ \dot{P} = -sP + abVP\,, \end{cases}$$

where all parameters are positive and V and P reflect the size of prey and predator populations, respectively.

From the lecture we know that the normal L-V is structurally unstable, i.e we should see periodicity in the above system.

For 'infinite' K we are in the L-V regime.

Excercise 6 - logistic L-V with large carrying capacity

Implement MATLAB function solving the logistic L-V system

$$\begin{cases} \dot{V} = rV\left(1 - \frac{V}{K}\right) - aVP\,, \\ \dot{P} = -sP + abVP\,, \end{cases}$$

and plotting its trajectory in the phase plane.

Look at the solution behaviour for large K and for $r = 1,\ a = 2,\ s = 1,\ b = 1.3$ and $(V(0, P(0)) = (0.25, 0.25)$.

Exercise 6 - solution

```
function logisticLV(par, init, T)

    opt=odeset('RelTol',1e-8, 'AbsTol',1e-8);
    sol = ode45(@model,[0 T],init,opt);

    figure(3)
    clf
    plot(sol.y(1,:),sol.y(2,:));

    function y=model(~,x,varargin)
      y=ones(2,1);
      y(1)=par.r*x(1)*(1-x(1)/par.K)-par.a*x(1)*x(2);
      y(2)=-par.s*x(2)+par.a*par.b*x(1)*x(2);
    end

end
```

Exercise 6 - solution