Students project 2014

Course: Mathematical Models of Biology and Medical Sciences

Task 1. On the basis of information presented in Section 1 propose a mathematical model expressed in the framework of ordinary differential equations.

Task 2. Perform mathematical analysis of the proposed model:

- existence and uniqueness of solutions;
- positivity of solutions for positive initial data;
- global existence of solutions in time;
- phase portraits;
- asymptotic behaviour of solutions.

Task 3. Estimate the model parameters on the basis of experimental data presented in Section 2, i.e. fit the model solution to the experimental data points.

1 Biological phenomena

Let us consider the interaction of immune system (effector cells) with the antigen. On the basis of biological experiments we know the following:

- The antigen reproduces on its own, but there exists a limiting value for its blood concentration.
- The effector cells are recruited only if the antigen is in the system.
- The more antigen is in the system the larger is the influx of the effector cells.
- The spontaneous death of effector cells is negligible.
- The antigen is removed from the system by interaction with effector cells.
- Some of the effector cells die after successful removal of the antigen.
- There is a limit on the number of the antigen that can be removed by the effector cell in a given period of time.

2 Experimental results

Experiments with different initial concentration of antigen (1, 5, 10 and 15 arbitrary units) were performed. No effector cells were present at the beginning of each experiment. Measurements of the antigen concentration (V) and effector cells concentration (E) were performed every 4 days for 20 days. Experimental data is presented in Table 1. and illustrated in Figure 1.

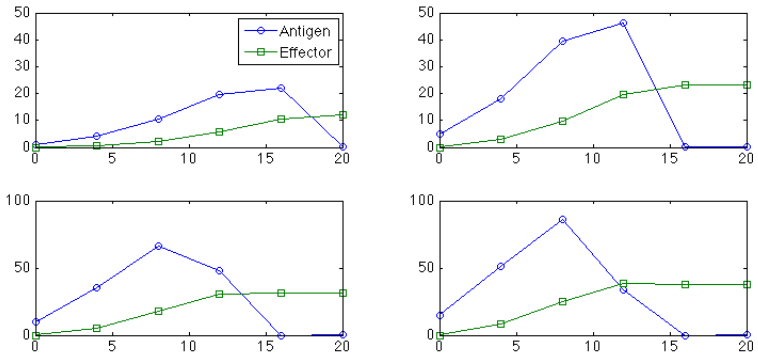

Figure 1: Plot of the experimental results.

Table 1: Experimental data.

	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
day	V	E	V	E	V	E	V	E
0	1	0	5	0	10	0	15	0
4	4	0.59	18	2.8	35	5.5	51	8.1
8	11	2.3	39	9.6	66	18	86	25
12	20	5.8	46	20	48	31	34	38
16	22	10	0.018	23	0	32	0	38
20	0.12	12	0	23	0	32	0	38